Review Exercises

In Exercises 1–4, sketch the graph of the quadratic function. Identify the vertex and the intercepts.

1. $f(x) = \left(x + \frac{3}{2}\right)^2 + 1$
2. $f(x) = (x - 4)^2 - 4$
3. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$
4. $f(x) = 3x^2 - 12x + 11$

In Exercises 5 and 6, find the quadratic function that has the indicated vertex and whose graph passes through the given point.

5. Vertex: $(1, -4)$; Point: $(2, -3)$
6. Vertex: $(2, 3)$; Point: $(-1, 6)$

In Exercises 7–12, find the maximum or minimum value of the quadratic function.

7. $g(x) = x^2 - 2x$
8. $f(x) = x^2 + 8x + 10$
9. $f(x) = 6x - x^2$
10. $h(x) = 3 + 4x - x^2$
11. $f(t) = -2t^2 + 4t + 1$
12. $h(x) = 4x^2 + 4x + 13$

13. *Numerical, Graphical, and Analytical Analysis* A rectangle is inscribed in the region bounded by the x-axis, the y-axis, and the graph of $x + 2y - 8 = 0$ (see figure).

(a) Complete six rows of a table such as this one.

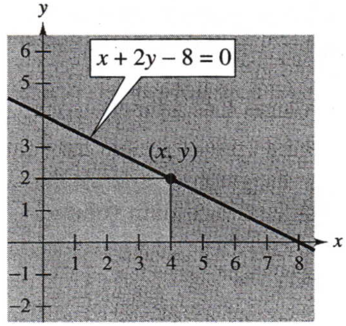
x	y	Area
1	$4 - \frac{1}{2}(1)$	$(1)[4 - \frac{1}{2}(1)] = \frac{7}{2}$
2	$4 - \frac{1}{2}(2)$	$(2)[4 - \frac{1}{2}(2)] = 6$

(b) Use a graphing utility to generate additional rows of the table. Use the table to estimate the dimensions that will produce the maximum area.

(c) Write the area A as a function of x. Determine the domain of the function in the context of the problem.

(d) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum area.

(e) Write the area function in standard form to find analytically the dimensions that will produce the maximum area.



14. *Maximum Profit* Let x be the amount (in hundreds of dollars) a company spends on advertising, and let P be the profit, where

$$P = 230 + 20x - \frac{1}{2}x^2.$$

What amount of advertising will yield a maximum profit?

In Exercises 15–18, determine the right-hand and left-hand behavior of the graph of the polynomial function.

15. $f(x) = -x^2 + 6x + 9$
16. $f(x) = \frac{1}{2}x^3 + 2x$
17. $g(x) = \frac{3}{4}(x^4 + 3x^2 + 2)$
18. $h(x) = -x^5 - 7x^2 + 10x$

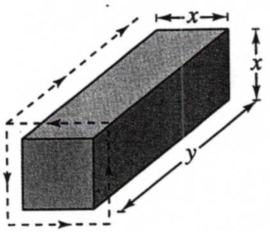
Graphical Analysis **In Exercises 19 and 20, use a graphing utility to graph the functions f and g on the same viewing rectangle. Zoom out sufficiently far to show that the right-hand and left-hand behavior of f and g appear identical.**

19. $f(x) = \frac{1}{2}x^3 - 2x + 1, \quad g(x) = \frac{1}{2}x^3$
20. $f(x) = -x^4 + 2x^3, \quad g(x) = -x^4$

In Exercises 21–26, sketch the graph of the function.

21. $g(x) = x^4 - x^3 - 2x^2$
22. $h(x) = -2x^3 - x^2 + x$
23. $f(t) = t^3 - 3t$
24. $f(x) = -x^3 + 3x - 2$
25. $f(x) = x(x + 3)^2$
26. $f(t) = t^4 - 4t^2$

27. ***Volume*** A rectangular package can have a maximum combined length and girth (perimeter of a cross section) of 216 centimeters (see figure).
    (a) Write the volume V as a function of x.
    (b) Use a graphing utility to graph the volume function, and use the graph to estimate the dimensions of the package of maximum volume.



28. ***Volume*** Rework Exercise 35 for a cylindrical package. (The cross sections are circular.)

Graphical Analysis **In Exercises 29 and 30, use a graphing utility to graph the two equations on the same viewing rectangle. Use the graphs to verify that the expressions are equivalent. Verify the results analytically.**

29. $y_1 = \dfrac{x^2}{x - 2},\quad y_2 = x + 2 + \dfrac{4}{x - 2}$
30. $y_1 = \dfrac{x^4 + 1}{x^2 + 2},\quad y_2 = x^2 - 2 + \dfrac{5}{x^2 + 2}$

In Exercises 31–36, perform the division.

31. $\dfrac{24x^2 - x - 8}{3x - 2}$
32. $\dfrac{4x + 7}{3x - 2}$
33. $\dfrac{x^4 - 3x^2 + 2}{x^2 - 1}$
34. $\dfrac{3x^4}{x^2 - 1}$
35. $\dfrac{x^4 + x^3 - x^2 + 2x}{x^2 + 2x}$
36. $\dfrac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1}$

In Exercises 37–40, use synthetic division to perform the division.

37. $(0.25x^4 - 4x^3) \div (x - 2)$
38. $(2x^3 + 2x^2 - x + 2) \div \left(x - \frac{1}{2}\right)$
39. $(6x^4 - 4x^3 - 27x^2 + 18x) \div \left(x - \frac{2}{3}\right)$
40. $(0.1x^3 + 0.3x^2 - 0.5) \div (x - 5)$

In Exercises 41 and 42, use synthetic division to decide whether the x-values are zeros of the function.

41. $f(x) = 2x^3 + 3x^2 - 20x - 21$
    (a) $x = 4$ (b) $x = -1$
    (c) $x = -\frac{7}{2}$ (d) $x = 0$
42. $f(x) = 20x^4 + 9x^3 - 14x^2 - 3x$
    (a) $x = -1$ (b) $x = \frac{3}{4}$
    (c) $x = 0$ (d) $x = 1$

In Exercises 43–48, perform the operations and write the result in standard form.

43. $(7 + 5i) + (-4 + 2i)$
44. $\left(\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i\right) - \left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)$
45. $5i(13 - 8i)$
46. $i(6 + i)(3 - 2i)$
47. $\dfrac{6 + i}{i}$
48. $\dfrac{3 + 2i}{5 + i}$

In Exercises 49 and 50, find a polynomial with integer coefficients that has the given zeros.

49. $-1, -1, \frac{1}{3}, -\frac{1}{2}$
50. $2, -3, 1 - 2i, 1 + 2i$

In Exercises 51–56, find all the zeros of the function.

51. $f(x) = 4x^3 - 11x^2 + 10x - 3$
52. $f(x) = 10x^3 + 21x^2 - x - 6$
53. $f(x) = 6x^3 - 5x^2 + 24x - 20$
54. $f(x) = x^3 - 1.3x^2 - 1.7x + 0.6$
55. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$
56. $f(x) = 5x^4 + 126x^2 + 25$

In Exercises 57–60, use a graphing utility to (a) graph the function, (b) determine the number of real zeros of the function, and (c) approximate the real zeros of the function to the nearest hundredth.

57. $f(x) = x^4 + 2x + 1$
58. $g(x) = x^3 - 3x^2 + 3x + 2$
59. $h(x) = x^3 - 6x^2 + 12x - 10$
60. $f(x) = x^5 + 2x^3 - 3x - 20$

61. *Data Analysis* Sales (in billions of dollars) of recreational vehicles in the United States for the years 1980 through 1993 were as follows: 1980 (1.2), 1981 (1.8), 1982 (1.7), 1983(3.4), 1984 (4.1), 1985 (3.5), 1986 (3.9), 1987 (4.5), 1988 (4.8), 1989 (4.5), 1990 (4.1), 1991 (3.6), 1992 (4.4), 1993 (4.8).

    A model for the data is

    $S = 1.209 + 0.290t + 0.176t^2 - 0.031t^3 + 0.0013t^4$

    where S is the sales in billions of dollars and t is the time in years, with $t = 0$ corresponding to 1980. (Source: National Sporting Goods Association)

    (a) Use a graphing utility to plot the data points and graph the model.

    (b) The data shows that the sales were down from 1989 through 1991. Give a possible explanation. Does the model show the downturn in sales?

    (c) Use a graphing utility to approximate the magnitude of the decrease in sales during the slump described in part (b). Was the actual decrease more or less than indicated by the model?

    (d) Use the model to estimate sales in 1995.

62. *Age of the Groom* The average age of the groom at a wedding for a given age of the bride can be approximated by the model

    $$y = -0.00428x^2 + 1.442x - 3.136, \quad 20 \le x \le 55$$

    where y is the age of the groom and x is the age of the bride. For what age of the bride is the average age of the groom 30? (Source: U.S. National Center for Health Statistics)

In Exercises 63–70, sketch the graph of the rational function. As sketching aids, check for intercepts, symmetry, vertical asymptotes, horizontal asymptotes, and slant asymptotes.

63. $f(x) = \dfrac{-5}{x^2}$
64. $h(x) = \dfrac{x-3}{x-2}$
65. $P(x) = \dfrac{x^2}{x^2+1}$
66. $f(x) = \dfrac{2x}{x^2+4}$
67. $f(x) = \dfrac{x}{x^2+1}$
68. $h(x) = \dfrac{4}{(x-1)^2}$
69. $f(x) = \dfrac{2x^3}{x^2+1}$
70. $y = \dfrac{2x^2}{x^2-4}$

In Exercises 71–74, use a graphing utility to graph the function. Identify any vertical, horizontal, or slant asymptotes.

71. $s(x) = \dfrac{8x^2}{x^2+4}$
72. $y = \dfrac{5x}{x^2-4}$
73. $g(x) = \dfrac{x^2+1}{x+1}$
74. $y = \dfrac{1}{x+3} + 2$

In Exercises 75–80, write the partial fraction decomposition for the rational expression.

75. $\dfrac{4-x}{x^2+6x+8}$
76. $\dfrac{-x}{x^2+3x+2}$
77. $\dfrac{x^2}{x^2+2x-15}$
78. $\dfrac{9}{x^2-9}$
79. $\dfrac{x^2+2x}{x^3-x^2+x-1}$
80. $\dfrac{4x-2}{3(x-1)^2}$